MMAT5010 Linear Analysis (2024-25): Homework 4
Deadline: 1 Mar 2025

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Suppose that the space $\mathbb{R}^2$ is endowed with the usual norm, that is $\|(x_1, x_2)\| := \sqrt{x_1^2 + x_2^2}$. Let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$$
Find $\|A\|$?

2. Let X be a normed space and let $V : X \to X$ be an isometric isomorphism on X, that is T is linear isomorphism and $\|Vx\| = \|x\|$ for all $x \in X$. Show that if $T \in L(X)$, then $\|VTV^{-1}\| = \|T\|$.

3. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$. For each element $(x, y) \in X \oplus Y$, put
$$\|(x, y)\|_1 := \|x\|_X + \|y\|_Y.$$
Let $T \in L(X)$ and $S \in L(Y)$. Define $T \oplus S : X \oplus Y \to X \oplus Y$ by $(T \oplus S)(x, y) := (Tx, Sy)$. Show that $\|T \oplus S\| = \max(\|T\|, \|S\|)$.

$* * *$ **End** $* * *$